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Electromagnetic Theory I

Transmission Lines

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Transmission Lines

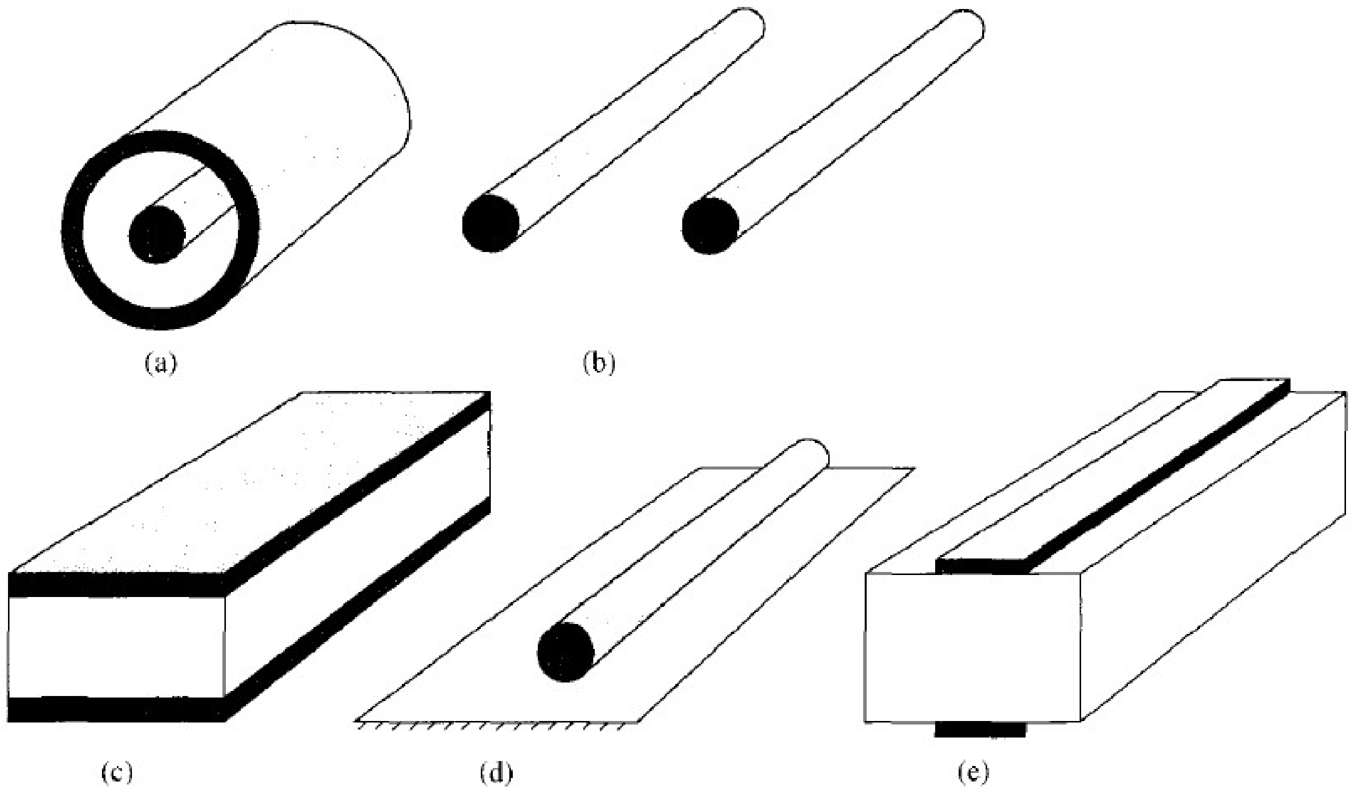

Figure 11.1 Cross-sectional view of typical transmission lines: **(a)** coaxial line, **(b)** two-wire line, **(c)** planar line, **(d)** wire above conducting plane, **(e)** microstrip line.

Transmission Lines Parameters

TABLE 11.1 Distributed Line Parameters at High Frequencies*

Parameters	Coaxial Line	Two-Wire Line	Planar Line
R (Ω/m)	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ $(\delta \ll a, c - b)$	$\frac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\frac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
L (H/m)	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
G (S/m)	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
C (F/m)	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ $(w \gg d)$

$*\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\frac{d}{2a} \simeq \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

$LC = \mu\varepsilon$ and $\frac{G}{C} = \frac{\sigma}{\varepsilon}$

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Transmission Lines Parameters

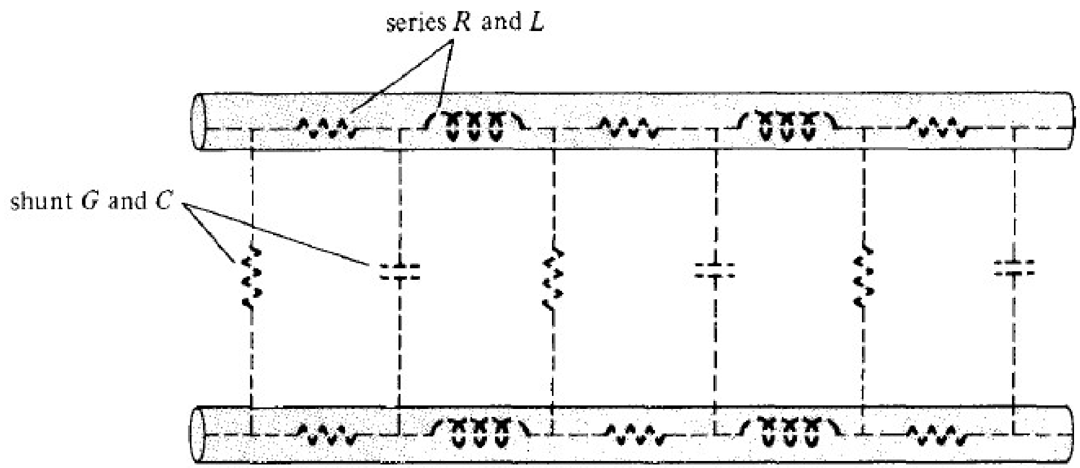

Figure 11.3 Distributed parameters of a two-conductor transmission line.

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Transmission Lines Parameters

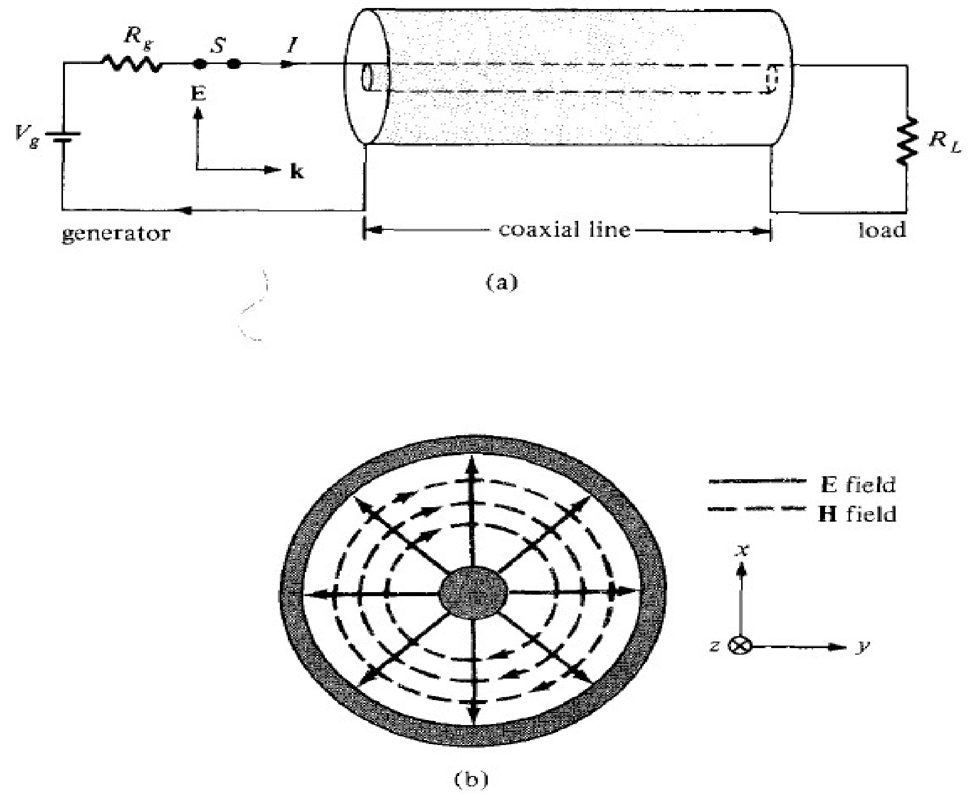

Figure 11.4 (a) Coaxial line connecting the generator to the load; **(b) E** and **H** fields on the coaxial line.

Transmission Lines Equations

An important property of TEM waves is that the fields **E** and **H** are uniquely related to voltage V and current I, respectively:

$$V = -\int \mathbf{E} \cdot d\mathbf{l}, \qquad I = \oint \mathbf{H} \cdot d\mathbf{l}$$

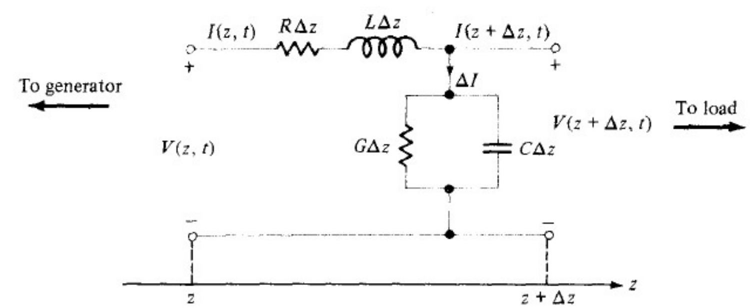

Figure 11.5 *L*-type equivalent circuit model of a differential length Δz of a two-conductor transmission line.

By applying Kirchhoff's voltage law to the outer loop of the circuit in Figure 11.5, we obtain

$$V(z, t) = R\,\Delta z\, I(z, t) + L\,\Delta z \frac{\partial I(z, t)}{\partial t} + V(z + \Delta z, t)$$

or

$$\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = R\, I(z, t) + L \frac{\partial I(z, t)}{\partial t} \qquad (11.3)$$

Transmission Lines Equations

Taking the limit of eq. (11.3) as $\Delta z \rightarrow 0$ leads to

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \tag{11.4}$$

Similarly, applying Kirchoff's current law to the main node of the circuit in Figure 11.5 gives

$$\begin{aligned} I(z,t) &= I(z+\Delta z,t) + \Delta I \\ &= I(z+\Delta z,t) + G\,\Delta z\,V(z+\Delta z,t) + C\,\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t} \end{aligned}$$

or

$$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = G\,V(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t} \tag{11.5}$$

As $\Delta z \rightarrow 0$, eq. (11.5) becomes

$$-\frac{\partial I(z,t)}{\partial z} = G\,V(z,t) + C\frac{\partial V(z,t)}{\partial t} \tag{11.6}$$

Transmission Lines Equations

If we assume harmonic time dependence so that

$$V(z, t) = \text{Re}\,[V_s(z)\, e^{j\omega t}] \quad (11.7a)$$

$$I(z, t) = \text{Re}\,[I_s(z)\, e^{j\omega t}] \quad (11.7b)$$

where $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z, t)$ and $I(z, t)$, respectively, eqs. (11.4) and (11.6) become

$$-\frac{dV_s}{dz} = (R + j\omega L)\, I_s \quad (11.8)$$

$$-\frac{dI_s}{dz} = (G + j\omega C)\, V_s \quad (11.9)$$

In the differential eqs. (11.8) and (11.9), V_s and I_s are coupled. To separate them, we take the second derivative of V_s in eq. (11.8) and employ eq. (11.9) so that we obtain

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)\, V_s$$

or

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0 \quad (11.10)$$

Transmission Lines Equations

$$\lambda = \frac{2\pi}{\beta}$$

$$u = \frac{\omega}{\beta} = f\lambda$$

$$V_s(z) = \underset{\longrightarrow\ +z}{V_o^+ e^{-\gamma z}} + \underset{-z\ \longleftarrow}{V_o^- e^{\gamma z}}$$

$$I_s(z) = \underset{\longrightarrow\ +z}{I_o^+ e^{-\gamma z}} + \underset{-z\ \longleftarrow}{I_o^- e^{\gamma z}}$$

$$\begin{aligned} V(z, t) &= \text{Re}\,[V_s(z)\, e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned}$$

Transmission Lines Equations

The **characteristic impedance** Z_o of the line is the ratio of positively traveling voltage wave to current wave at any point on the line.

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

or

$$\boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o}$$

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Transmission Lines Equations

A. Lossless Line ($R = 0 = G$)

A **transmission line** is said to be **lossless** if the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma \approx 0$).

For such a line, it is evident from Table 11.1 that when $\sigma_c \simeq \infty$ and $\sigma \simeq 0$.

$$\boxed{R = 0 = G} \tag{11.20}$$

This is a necessary condition for a line to be lossless. Thus for such a line, eq. (11.20) forces eqs. (11.11), (11.14), and (11.19) to become

$$\alpha = 0, \qquad \gamma = j\beta = j\omega\sqrt{LC} \tag{11.21a}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.21b}$$

$$X_o = 0, \qquad Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.21c}$$

Transmission Lines Equations

B. Distortionless Line ($R/L = G/C$)

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as α is frequency dependent. This results in distortion.

A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

From the general expression for α and β [in eq. (11.11)], a distortionless line results if the line parameters are such that

$$\boxed{\frac{R}{L} = \frac{G}{C}} \qquad (11.22)$$

Thus, for a distortionless line,

$$\begin{aligned}\gamma &= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)} \\ &= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta\end{aligned}$$

or

$$\alpha = \sqrt{RG}, \qquad \beta = \omega\sqrt{LC} \qquad (11.23a)$$

Transmission Lines Equations

showing that α does not depend on frequency whereas β is a linear function of frequency. Also

$$Z_o = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o + jX_o$$

or

$$R_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \qquad X_o = 0 \tag{11.23b}$$

and

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \tag{11.23c}$$

Transmission Lines Equations

Note that

1. The phase velocity is independent of frequency because the phase constant β linearly depends on frequency. We have shape distortion of signals unless α and u are independent of frequency.
2. u and Z_o remain the same as for lossless lines.
3. A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

TABLE 11.2 Transmission Line Characteristics

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic Impedance $Z_o = R_o + jX_o$
General	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$

Transmission Lines Equations

EXAMPLE 11.1 An air line has characteristic impedance of 70 Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

Solution:

An air line can be regarded as a lossless line since $\sigma \simeq 0$. Hence

$$R = 0 = G \qquad \text{and} \qquad \alpha = 0$$

$$Z_o = R_o = \sqrt{\frac{L}{C}} \tag{11.1.1}$$

$$\beta = \omega\sqrt{LC} \tag{11.1.2}$$

Dividing eq. (11.1.1) by eq. (11.1.2) yields

$$\frac{R_o}{\beta} = \frac{1}{\omega C}$$

or

$$C = \frac{\beta}{\omega R_o} = \frac{3}{2\pi \times 100 \times 10^6 (70)} = 68.2 \text{ pF/m}$$

From eq. (11.1.1),

$$L = R_o^2 C = (70)^2(68.2 \times 10^{-12}) = 334.2 \text{ nH/m}$$

Transmission Lines Equations

EXAMPLE 11.2

A distortionless line has $Z_o = 60\ \Omega$, $\alpha = 20$ mNp/m, $u = 0.6c$, where c is the speed of light in a vacuum. Find R, L, G, C, and λ at 100 MHz.

Solution:

For a distortionless line,

$$RC = GL \qquad \text{or} \qquad G = \frac{RC}{L}$$

and hence

$$Z_o = \sqrt{\frac{L}{C}} \tag{11.2.1}$$

$$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_o} \tag{11.2.2a}$$

or

$$R = \alpha Z_o \tag{11.2.2b}$$

But

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \tag{11.2.3}$$

Transmission Lines Equations

From eq. (11.2.2b),

$$R = \alpha Z_o = (20 \times 10^{-3})(60) = 1.2\ \Omega/\text{m}$$

Dividing eq. (11.2.1) by eq. (11.2.3) results in

$$L = \frac{Z_o}{u} = \frac{60}{0.6\,(3 \times 10^8)} = 333\ \text{nH/m}$$

From eq. (11.2.2a),

$$G = \frac{\alpha^2}{R} = \frac{400 \times 10^{-6}}{1.2} = 333\ \mu\text{S/m}$$

Multiplying eqs. (11.2.1) and (11.2.3) together gives

$$uZ_o = \frac{1}{C}$$

or

$$C = \frac{1}{uZ_o} = \frac{1}{0.6\,(3 \times 10^8)\,60} = 92.59\ \text{pF/m}$$

$$\lambda = \frac{u}{f} = \frac{0.6\,(3 \times 10^8)}{10^8} = 1.8\ \text{m}$$

INPUT IMPEDANCE, SWR, AND POWER

Let the transmission line extend from $z = 0$ at the generator to $z = \ell$ at the load. First of all, we need the voltage and current waves in eqs. (11.15) and (11.16), that is

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \tag{11.24}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \tag{11.25}$$

where eq. (11.18) has been incorporated. To find V_o^+ and V_o^-, the terminal conditions must be given. For example, if we are given the conditions at the input, say

$$V_o = V(z = 0), \qquad I_o = I(z = 0) \tag{11.26}$$

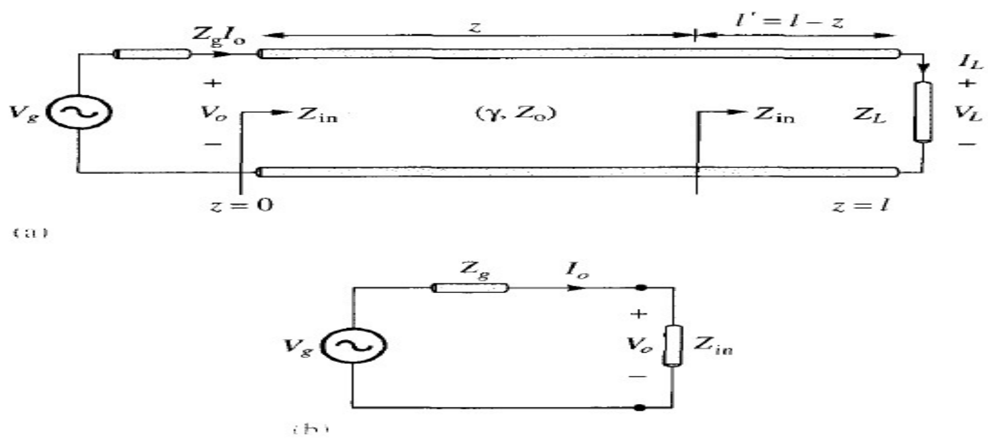

Figure 11.6 (a) Input impedance due to a line terminated by a load; (b) equivalent circuit for finding V_o and I_o in terms of Z_{in} at the input.

INPUT IMPEDANCE, SWR, AND POWER

substituting these into eqs. (11.24) and (11.25) results in

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o) \tag{11.27a}$$

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o) \tag{11.27b}$$

If the input impedance at the input terminals is Z_{in}, the input voltage V_o and the input current I_o are easily obtained from Figure 11.6(b) as

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g} \tag{11.28}$$

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = \ell), \qquad I_L = I(z = \ell) \tag{11.29}$$

Substituting these into eqs. (11.24) and (11.25) gives

$$V_o^+ = \frac{1}{2}(V_L + Z_o I_L)e^{\gamma\ell} \tag{11.30a}$$

$$V_o^- = \frac{1}{2}(V_L - Z_o I_L)e^{-\gamma\ell} \tag{11.30b}$$

INPUT IMPEDANCE, SWR, AND POWER

Next, we determine the input impedance $Z_{in} = V_s(z)/I_s(z)$ at any point on the line. At the generator, for example, eqs. (11.24) and (11.25) yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-} \tag{11.31}$$

Substituting eq. (11.30) into (11.31) and utilizing the fact that

$$\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} = \cosh \gamma\ell, \qquad \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} = \sinh \gamma\ell \tag{11.32a}$$

or

$$\tanh \gamma\ell = \frac{\sinh \gamma\ell}{\cosh \gamma\ell} = \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{e^{\gamma\ell} + e^{-\gamma\ell}} \tag{11.32b}$$

we get

$$\boxed{Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell} \right]} \qquad \text{(lossy)} \tag{11.33}$$

INPUT IMPEDANCE, SWR, AND POWER

Although eq. (11.33) has been derived for the input impedance Z_{in} at the generation end, it is a general expression for finding Z_{in} at any point on the line. To find Z_{in} at a distance ℓ' from the load as in Figure 11.6(a), we replace ℓ by ℓ'. A formula for calculating the hyperbolic tangent of a complex number, required in eq. (11.33), is found in Appendix A.3.

For a lossless line, $\gamma = j\beta$, $\tanh j\beta\ell = j\tan\beta\ell$, and $Z_o = R_o$, so eq. (11.33) becomes

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tan\beta\ell}{Z_o + jZ_L\tan\beta\ell}\right] \quad \text{(lossless)} \tag{11.34}$$

showing that the input impedance varies periodically with distance ℓ from the load. The quantity $\beta\ell$ in eq. (11.34) is usually referred to as the *electrical length* of the line and can be expressed in degrees or radians.

We now define Γ_L as the *voltage reflection coefficient* (at the load). Γ_L is the ratio of the voltage reflection wave to the incident wave at the load, that is,

$$\Gamma_L = \frac{V_o^- e^{\gamma\ell}}{V_o^+ e^{-\gamma\ell}} \tag{11.35}$$

Substituting V_o^- and V_o^+ in eq. (11.30) into eq. (11.35) and incorporating $V_L = Z_L I_L$ gives

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \tag{11.36}$$

INPUT IMPEDANCE, SWR, AND POWER

As a way of demonstrating these concepts, consider a lossless line with characteristic impedance of $Z_o = 50\ \Omega$. For the sake of simplicity, we assume that the line is terminated in a pure resistive load $Z_L = 100\ \Omega$ and the voltage at the load is 100 V (rms). The conditions on the line are displayed in Figure 11.7. Note from the figure that conditions on the line repeat themselves every half wavelength.

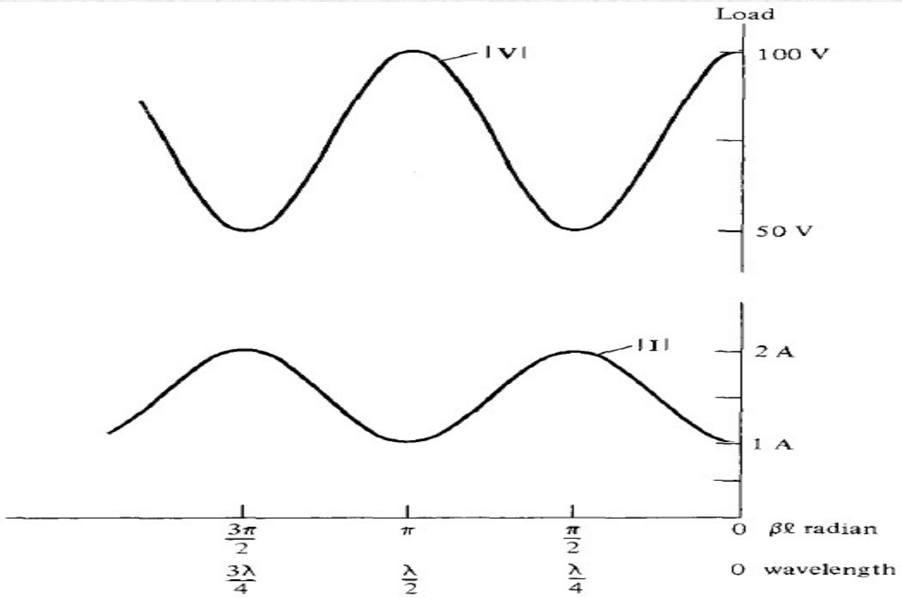

Figure 11.7 Voltage and current wave patterns on a lossless line terminated by a resistive load.

INPUT IMPEDANCE, SWR, AND POWER

$$P_{ave} = \frac{1}{2} \text{Re} \left[V_s(\ell) I_s^*(\ell) \right]$$

where the factor $\frac{1}{2}$ is needed since we are dealing with the peak values instead of the rms values. Assuming a lossless line, we substitute eqs. (11.24) and (11.25) to obtain

$$P_{ave} = \frac{1}{2} \text{Re} \left[V_o^+ (e^{j\beta\ell} + \Gamma e^{-j\beta\ell}) \frac{V^{+*}}{Z_o} (e^{-j\beta\ell} - \Gamma^* e^{j\beta\ell}) \right]$$

$$= \frac{1}{2} \text{Re} \left[\frac{|V_o^+|^2}{Z_o} (1 - |\Gamma|^2 + \Gamma e^{-2j\beta\ell} - \Gamma^* e^{2j\beta\ell}) \right]$$

Since the last two terms are purely imaginary, we have

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o} (1 - |\Gamma|^2) \qquad (11.40)$$

The first term is the incident power P_i, while the second term is the reflected power P_r. Thus eq. (11.40) may be written as

$$P_t = P_i - P_r$$

where P_t is the input or transmitted power and the negative sign is due to the negative-going wave since we take the reference direction as that of the voltage/current traveling toward the right. We should notice from eq. (11.40) that the power is constant and does not depend on ℓ since it is a lossless line. Also, we should notice that maximum power is delivered to the load when $\Gamma = 0$, as expected.

We now consider special cases when the line is connected to load $Z_L = 0$, $Z_L = \infty$, and $Z_L = Z_o$. These special cases can easily be derived from the general case.

INPUT IMPEDANCE, SWR, AND POWER

A. Shorted Line ($Z_L = 0$)

For this case, eq. (11.34) becomes

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_o \tan \beta\ell \tag{11.41a}$$

Also,

$$\Gamma_L = -1, \qquad s = \infty \tag{11.41b}$$

We notice from eq. (11.41a) that Z_{in} is a pure reactance, which could be capacitive or inductive depending on the value of ℓ. The variation of Z_{in} with ℓ is shown in Figure 11.8(a).

B. Open-Circuited Line ($Z_L = \infty$)

In this case, eq. (11.34) becomes

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_o}{j \tan \beta\ell} = -jZ_o \cot \beta\ell \tag{11.42a}$$

and

$$\Gamma_L = 1, \qquad s = \infty \tag{11.42b}$$

The variation of Z_{in} with ℓ is shown in Figure 11.8(b). Notice from eqs. (11.41a) and (11.42a) that

$$Z_{sc}Z_{oc} = Z_o^2 \tag{11.43}$$

INPUT IMPEDANCE, SWR, AND POWER

C. Matched Line ($Z_L = Z_o$)

This is the most desired case from the practical point of view. For this case, eq. (11.34) reduces to

$$Z_{in} = Z_o \tag{11.44a}$$

and

$$\Gamma_L = 0, \qquad s = 1 \tag{11.44b}$$

INPUT IMPEDANCE, SWR, AND POWER

EXAMPLE 11.3 A certain transmission line operating at $\omega = 10^6$ rad/s has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_o = 60 + j40\ \Omega$, and is 2 m long. If the line is connected to a source of $10\underline{/0^\circ}$ V, $Z_g = 40\ \Omega$ and terminated by a load of $20 + j50\ \Omega$, determine

(a) The input impedance

(b) The sending-end current

(c) The current at the middle of the line

Solution:

(a) Since 1 Np = 8.686 dB,

$$\alpha = \frac{8}{8.686} = 0.921 \text{ Np/m}$$

$$\gamma = \alpha + j\beta = 0.921 + j1 \text{ /m}$$

$$\gamma\ell = 2(0.921 + j1) = 1.84 + j2$$

Using the formula for $\tanh(x + jy)$ in Appendix A.3, we obtain

$$\tanh \gamma\ell = 1.033 - j0.03929$$

$$Z_{in} = Z_o\left(\frac{Z_L + Z_o \tanh \gamma\ell}{Z_o + Z_L \tanh \gamma\ell}\right)$$

$$= (60 + j40)\left[\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)}\right]$$

$$Z_{in} = 60.25 + j38.79\ \Omega$$

(b) The sending-end current is $I(z = 0) = I_o$. From eq. (11.28),

$$I(z = 0) = \frac{V_g}{Z_{in} + Z_g} = \frac{10}{60.25 + j38.79 + 40}$$

$$= 93.03\underline{/-21.15^\circ} \text{ mA}$$

INPUT IMPEDANCE, SWR, AND POWER

(c) To find the current at any point, we need V_o^+ and V_o^-. But

$$I_o = I(z = 0) = 93.03\angle -21.15^\circ \text{ mA}$$

$$V_o = Z_{in} I_o = (71.66\angle 32.77^\circ)(0.09303\angle -21.15^\circ) = 6.667\angle 11.62^\circ \text{ V}$$

From eq. (11.27),

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o)$$
$$= \frac{1}{2}[6.667\angle 11.62^\circ + (60 + j40)(0.09303\angle -21.15^\circ)] = 6.687\angle 12.08^\circ$$

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o) = 0.0518\angle 260^\circ$$

At the middle of the line, $z = \ell/2$, $\gamma z = 0.921 + j1$. Hence, the current at this point is

$$I_s(z = \ell/2) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$
$$= \frac{(6.687e^{j12.08^\circ})e^{-0.921 - j1}}{60 + j40} - \frac{(0.0518e^{j260^\circ})e^{0.921 + j1}}{60 + j40}$$

INPUT IMPEDANCE, SWR, AND POWER

Note that $j1$ is in radians and is equivalent to $j57.3°$. Thus,

$$\begin{aligned} I_s(z = \ell/2) &= \frac{6.687e^{j12.08°}e^{-0.921}e^{-j57.3°}}{72.1e^{j33.69°}} - \frac{0.0518e^{j260°}e^{0.921}e^{j57.3°}}{72.1e^{33.69°}} \\ &= 0.0369e^{-j78.91°} - 0.001805e^{j283.61°} \\ &= 6.673 - j34.456 \text{ mA} \\ &= 35.10\underline{/281°} \text{ mA} \end{aligned}$$